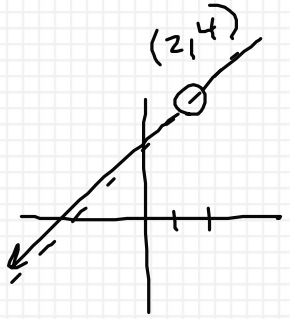
Chapter 2

Limits

2.1 – Limits as $x \to a$ (a is constant)

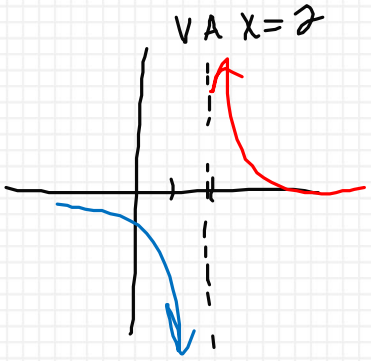
$\lim_{x \to 2}(x^2 - 4) = 2^2 - 4 = \boxed{0}$



$\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2} = \dfrac{(x-2)(x+2)}{x-2} = 2+2 = \boxed{4}$

$\lim_{x \to 2} \dfrac{1}{x - 2}$

DNE



$f(2.1) = \dfrac{1}{2.1-2} = \dfrac{1}{.1} = 10$

$f(1.9) = \dfrac{1}{1.9-2} = \dfrac{1}{-.1} = -10$

1. Plug in a.
2. Factor, cancel, plug in a.
3. Consider the graph – most likely there is a vert. asymptote at $x = a$. Answer will be ∞, $-\infty$ or DNE.

2.1 – Special Trig Limits

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$\lim_{x\to 0}\frac{\sin 7x}{7x} = 7$$

$$\lim_{x\to 0}\frac{\sin 5x}{\sin 2x} = \frac{5}{2}$$

$$\lim_{x\to 0}\frac{\sin^2 x}{x}$$

$$\lim_{x\to 0}\frac{5x\cdot\frac{\sin 5x}{5x}}{2x\cdot\frac{\sin 2x}{2x}}$$

$$\lim_{x\to 0}\frac{\sin x\cdot\sin x}{x}$$

$\sin(0)$

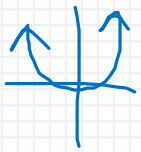
0

2.2 – Limits as $x \to \pm\infty$

Polynomial Functions: CONSIDER THE PARENT GRAPH | FUNCTION

$\lim_{x \to -\infty} 4x^4 - 3x^2 + x = \infty$

$4(-\infty)^4 = \infty$



2.2 – Limits as $x \to \pm\infty$

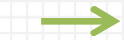
$h(x) = \frac{g(x)}{f(x)}$ $g(x), f(x)$ ARE POLYNOMIALS

HORIZONTAL ASYMP. @ $y = 0$

Rational Functions: degree of denominator > degree of numerator

$$\lim_{x\to\infty} \frac{4x^2 - 3x + 1}{x^3 + 2x} = \lim_{x\to\infty} \frac{4}{x} \to 0$$

$$\lim_{x\to\pm\infty} h(x) = 0$$



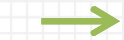
2.2 – Limits as $x \to \pm\infty$

HORIZONTAL ASYMPTOTE

Rational Functions: degree of denominator = degree of numerator

$$\lim_{x\to-\infty} \frac{4x^2+3x-1}{2x^2+1} = \lim_{x\to-\infty} \frac{4x^2}{2x^2} = \lim_{x\to-\infty} 2 = \boxed{2}$$

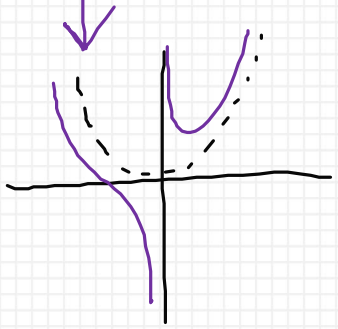
$\lim_{x\to\pm\infty} h(x) =$ RATIO OF LEADING COEFFICIENTS



2.2 – Limits as $x \to \pm\infty$

Rational Functions: degree of denominator < degree of numerator

$$\lim_{x\to\infty} \frac{4x^3+3x+1}{2x+1} = \lim_{x\to\infty} \frac{4x^3}{2x} = \lim_{x\to\infty} 2x^2 = \infty$$



REDUCE TO A POLYNOMIAL

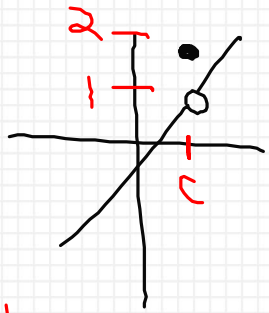
ANSWER IS $\infty, -\infty$, OR DNE

2.3 – Continuity

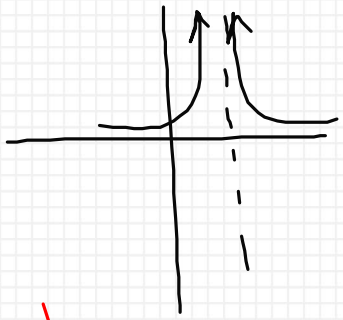
Definition of Continuity at a Point:

$f(x)$ is cont @ $x=c$ IFF $\lim_{x \to c} f(x) = f(c)$

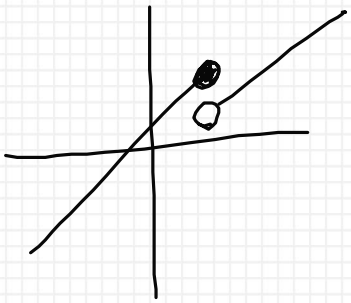
Types of Discontinuities:



$\lim_{x \to c} f(x) \neq f(c)$



$\lim_{x \to c}$ DNE



$\lim_{x \to c}$ DNE

Homework:

AP Packet #17 – 23

p. 91 #1 – 19 odd, 25, 29, 31, 39